

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name : Engineering Mathematics – I

Subject Code : 4TE01EMT2

Branch: B.Tech (All)

Semester : 1

Date : 21/03/2018

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

- a) If $y = (10)^{2x}$ then y_n equal to
 (A) $2^n (10)^{2x} (\log 10)^n$ (B) $2^n (10)^{2x} (\log 2)^n$ (C) $2^n (10)^{2x} (\log 10)^{n+1}$
 (D) none of these
- b) If $y = \cosh 2x$ and if n is even, then y_n equal to
 (A) $2^n \sinh 2x$ (B) $2^n \cosh 2x$ (C) $\cosh(2nx)$ (D) none of these
- c) If $y = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots$ then x equal to
 (A) $y - \frac{y^2}{2!} + \frac{y^3}{3!} - \frac{y^4}{4!} + \dots$ (B) $y - \frac{y^3}{3} + \frac{y^5}{5} - \dots$ (C) $y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \dots$
 (D) none of these
- d) The series $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$ represent expansion of
 (A) $\cot^{-1} x$ (B) $\tan^{-1} x$ (C) $\sin^{-1} x$ (D) $\sin x$
- e) $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x}{2} \right)^{\frac{1}{x}} = \underline{\hspace{2cm}}$
 (A) $\log ab$ (B) \sqrt{ab} (C) $\frac{1}{2} \log ab$ (D) none of these
- f) $\lim_{x \rightarrow \infty} x^2 e^{-x} = \underline{\hspace{2cm}}$
 (A) -1 (B) 0 (C) 1 (D) none of these
- g) If $P = r \tan \theta$, then $\frac{\partial P}{\partial r}$ is equal to
 (A) $\sec^2 \theta$ (B) $\tan \theta$ (C) $\tan \theta + r \sec^2 \theta$ (D) $\frac{1}{2} \tan \theta$



- h) If $u(x, y, z) = 0$ then the value of $\frac{\partial x}{\partial y} \cdot \frac{\partial y}{\partial z} \cdot \frac{\partial z}{\partial x}$ is equal to
 (A) 1 (B) -1 (C) 0 (D) none of these
- i) The value of $(1+i)^5 \times (1-i)^5$ is
 (A) -8 (B) $8i$ (C) 8 (D) 32
- j) The polar form of the complex number $\frac{1+i}{1-i}$ is
 (A) $\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$ (B) $\sin \frac{\pi}{2} + i \cos \frac{\pi}{2}$ (C) $\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}$
 (D) $\sin \frac{\pi}{4} + i \cos \frac{\pi}{4}$
- k) If $z = re^{i\theta}$, then $|e^{iz}|$ is equal to
 (A) $e^{r \sin \theta}$ (B) $e^{-r \sin \theta}$ (C) $e^{-r \cos \theta}$ (D) $e^{r \cos \theta}$
- l) If the system of equations $x - ky - z = 0$, $kx - y - z = 0$, $x + y - z = 0$ has a non-zero solution, then value of k can be
 (A) -3 (B) $-\frac{1}{2}$ (C) 1 (D) 2
- m) An eigenvalue of a square matrix A is $\lambda = 0$. Then
 (A) $|A| \neq 0$ (B) A is symmetric (C) A is singular (D) A is skew-symmetric
- n) If every minor of order r of a matrix A is zero, then rank of A is
 (A) greater than r (B) equal to r (C) less than or equal to r
 (D) less than r

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a) If $y = a \cos(\log x) + b \sin(\log x)$ then prove that (5)

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0.$$

b) If $y = \frac{1}{x^2 + a^2}$ then find y_n . (5)

c) Expand $f(x) = x^4 - 11x^3 + 43x^2 - 60x + 14$ in powers of $(x-3)$. (4)

Q-3 Attempt all questions (14)

a) Expand $f(x) = \sec x$ in powers of x up to x^4 by Maclaurin's series. (5)

b) Prove that $\cos^{-1}[\tanh(\log x)] = \pi - 2\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \dots\right)$. (5)

c) If $y = \sin^4 x$ then find y_n . (4)

Q-4 Attempt all questions (14)

a) Evaluate: $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x + d^x}{4} \right)^{\frac{1}{x}}$ (5)

b) If $u = f(r)$ and $r^2 = x^2 + y^2$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$. (5)



c) If $z = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$ then show that $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0$. (4)

Q-5 Attempt all questions (14)

a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$. (5)

b) Evaluate: $\lim_{x \rightarrow 1} \left(\frac{x}{x-1} - \frac{1}{\log x} \right)$ (5)

c) Evaluate: $\lim_{x \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + x^2}{x^3}$ (4)

Q-6 Attempt all questions (14)

a) Solve the equation $z^3 = i(z-1)^3$. (5)

b) Using De Moivre's theorem prove the following: (5)

$$\cos 5\theta = 5\cos\theta - 20\cos^3\theta + 16\cos^5\theta$$

c) Check whether the following set of vectors is linearly dependent or linearly independent: (4)

$$(1, 2, -1, 0), (1, 3, 1, 2), (4, 2, 1, 0), (6, 1, 0, 1)$$

Q-7 Attempt all questions (14)

a) Examine for consistency and if consistent solve them (5)

$$x + 2y = 3, y - z = 2, x + y + z = 1$$

b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$. (5)

c) If $\sin(\alpha + i\beta) = x + iy$ then prove that $x^2 \cos^2 \alpha - y^2 \sec^2 \alpha = 1$. (4)

Q-8 Attempt all questions (14)

a) If $x_r = \cos \frac{\pi}{2^r} + i \sin \frac{\pi}{2^r}$ then prove that $\lim_{n \rightarrow \infty} x_1 x_2 x_3 \dots x_n = -1$. (5)

b) Reduce the matrix $A = \begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ to the normal form and find its rank. (5)

c) Discuss the continuity of the function (4)

$$f(x, y) = \frac{x}{\sqrt{x^2 + y^2}} \text{ when } (x, y) \neq (0, 0) \text{ and } f(x, y) = 2 \text{ when } (x, y) = (0, 0).$$

